

$$\text{target} = T \cdot \overbrace{D}^{\text{lengths}}$$

$$\text{start} = S$$

(INPUTS)

$$\text{right} = R \cdot W$$

$$\text{end} = E$$

$$U_p = \underbrace{U \cdot H}_{\text{unit vectors}}$$

$$n = S \cdot \frac{T}{\|T\|} \quad (z_{\text{Near}})$$

(SVN)

$$f = E \cdot \frac{T}{\|T\|} \quad (z_{\text{Far}})$$

$$q = \frac{n+f}{f} \quad (z_{\text{Scale}})$$

initial matrix * points P yields:

$$x = \frac{D}{2W} R \cdot P$$

$$y = -\frac{D}{2H} U \cdot P$$

$$z = (T \cdot P - n) q$$

$$w = T \cdot P \quad \leftarrow \text{(divisor for } x, y)$$

then adjust x, y:

$$x += \alpha w$$

$$\alpha \text{ (ofs 0)}$$

$$y += \beta w$$

$$\beta \text{ (ofs 1)}$$

such that:

$$\text{target} \rightarrow \left(\underbrace{\frac{1}{2}}_{\hat{x}}, \underbrace{\frac{1}{2}}_{\hat{y}}, ? \right)$$

full transform in 3 steps:

$$\begin{pmatrix} x \\ y \\ \textcircled{w} \end{pmatrix} = \begin{bmatrix} 1 & 0 & \alpha \\ 0 & 1 & \beta \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc|ccc} \frac{D}{2w} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{D}{2h} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \begin{bmatrix} R & 0 \\ u & 0 \\ T & 0 \\ \hline 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{pmatrix}$$

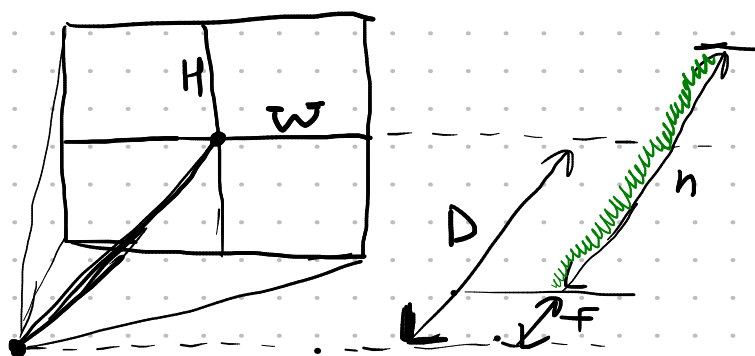
result
shift III
projection II
rotation I
input point

$$\frac{z}{h+f} = (T \cdot P - u) \frac{q}{h+f} = \frac{T \cdot P - u}{f} = \frac{T \cdot (P - s)}{T \cdot E}$$

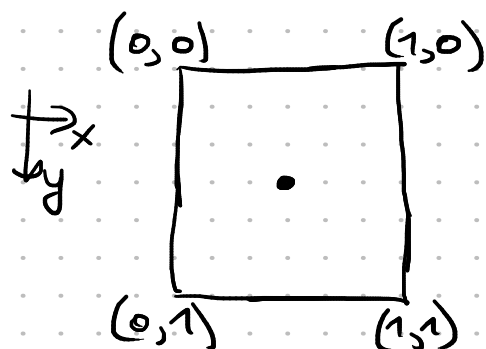
not divided by w!

$$\begin{aligned}
 \frac{z}{h+f} &= 0 \quad \text{at } P = S \\
 \frac{z}{h+f} &= 1 \quad \text{at } P = S + E
 \end{aligned}$$

linear



(after I)



(after I, II, III)

along z:

